Basics of Superluminal Signals

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Abstract. The paper elucidates the physical basis of experimental results on superluminal signal velocity. It will be made plausible that superluminal signals do not violate the principle of causality but they can shorten the luminal vacuum time span between cause and effect. This amazing behaviour is based on the property that any physical signal has a finite duration.

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1 Introduction

One of the first investigations on wave propagation was presented by Lord Rayleigh at the end of the 19th century. He considered that the group velocity corresponds to the velocity of energy or signal. This however raised difficulties with the theory of relativity in dispersive media. The problem was resolved by Sommerfeld and Brillouin in the case of waves with real wave numbers, however, it was not tackled for signals composed of evanescent modes only [1, 2]. For instance in a textbook on *Relativity, Groups, Particles* published recently a chapter is devoted to signal velocities *faster than light* [3]. The authors conclude that superluminal group velocities may have been measured, however, they state *the initial packet gets completely deformed and unsuitable for perfect signal transmission during the course of propagation due to the vastly differing phase velocities of its various frequency components.* The authors claim that a signal velocity never ever can exceed the vacuum velocity of light. Allegedly this has been proven by Einstein studying the light propagation in vacuum a hundred years ago in his famous paper [4].

In the case of tunnelling the above cited statement *vastly different phase velocity.....* is incorrect since an evanescent (tunnelling) mode has a purely imaginary wave number and does not experience a phase change [5, 6, 7, 8]. Therefore the phase does not depend on the field spreading inside a barrier. This outstanding property of the tunnelling process can indeed result in a superluminal signal velocity as will be reported in this article [8, 9]. Other books and papers even deny absolutely the possibility of either superluminal energy or signal velocities, see for instance references [10, 11, 12, 13]. Some authors claim that *frequency-band limitation is of course a technical and not a fundamental limitation* [14]. Yes certainly, there is a finite probability for higher frequency components, even if their energy $\hbar\omega$ exceeds tremendously the signal's total energy [13]. However, these high frequency components having an extremely low probability and short lifetime are not traceable at all and their statistical nature make fluctuations not suitable for signalling. Technical signals have usually a frequency band width much less than 1 % of the carrier frequency thus dispersion effects as a result of an interaction between the electromagnetic wave and any potential don't necessarily cause significant signal reshaping. For the extensive discussion and the various definitions of a signal see e.g. Refs. [9, 15, 16, 17]. For example a digital signal as displayed in Fig. 2a is defined by the envelope which contains the total information. The amplitude don't carry information only the half width represents the number of digits. Here the carrier frequency is $2{\times}10^{14}$ Hz and the relative frequency band modulation is 10^{-4}. For lang distance signal transmission signals were modulated on a high frequency carrier.

Figure 1 presents two measured superluminal signals. They are pulse-like shaped as we are used to from digital communication systems. One pulse has travelled at a negative group velocity of v_g = -c/310 the other one has tunnelled a photonic barrier at a speed of $v_g = 4.7\ c$, where c is the velocity of light in vacuum. The reader should inspect the two superluminal signals whether they are *completely deformed* as claimed in Ref. [3]. A deformation of the signals is not seen and both will be detected and interpreted properly as a well defined signal by the detector in question. In digital signal transmission the half width of the pulse represents the information (see Fig. 1 right and Fig. 2(a)). The superluminal time shift of the signal's envelope measured in both experiments represents the information that a superluminal signal including its energy have been measured. The signals shown in Fig. 1 have a frequency–band width of 120 kHz at a carrier frequency of 350 THz and 300 MHz at a carrier frequency of 10 GHz, respectively.

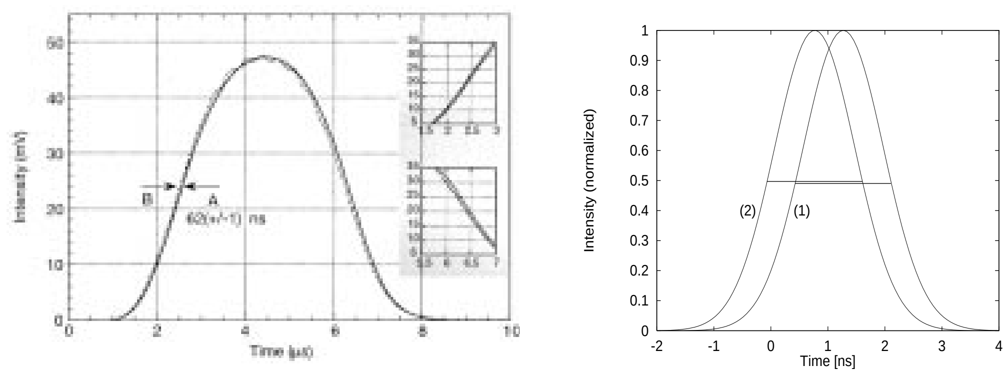
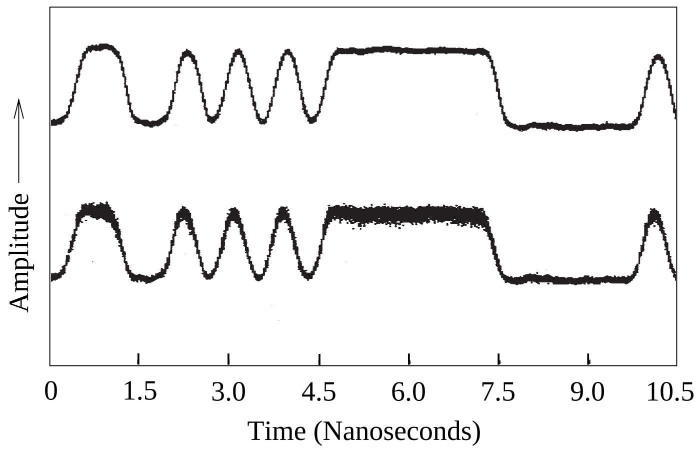


Fig. 1 Display of superluminal gain–assisted optical (left [18]) and tunnelled microwave pulses (right [19]). The pulses (= signals) are normalized (a signal does not depend on its magnitude) and compared with the air born A or the wave–guided ones (1). The measured time shifts of 62 ns and of 0.4 ns result in signal velocities of $-c/310$ and of $4.7\ c$, respectively.

Obviously superluminal signal velocities do exist. As an example of superluminal signal transmission the tunnelling process is discussed here. Electromagnetic waves with a purely imaginary wave number are called evanescent modes. They play an

(a)



(b)

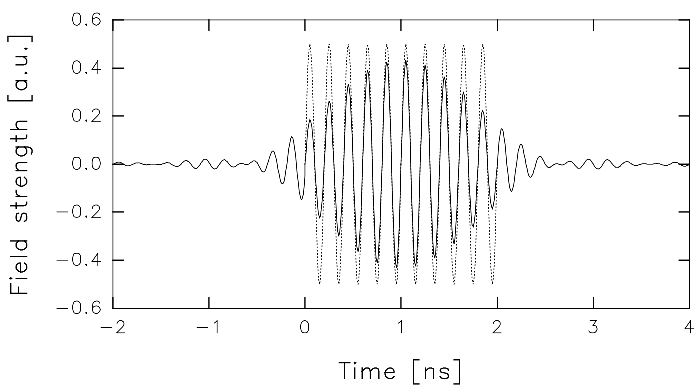


Fig. 2 Signals: (a) Measured fiber optical signal intensity in arbitrary units (a.u.). The half width corresponds to the number of bits: 1,1,0,0,1,0,1,0,1..... The lower signal was recorded after a distance of 9 000 km. The infrared carrier frequency of the signal is $2 \cdot 10^{14}$ Hz (wavelength 1.5μm). (b) Calculated sine wave signals; a mathematical non–frequency- (dotted line) and a physical frequency-band limited (solid line) [1]. As a result of the mathematical Fourier transform the frequency band limited signal has already components at negative times, i.e. before it is switched on. Such a noncausal behaviour, however, is not observed: Contrary to a mathematical signal a physical signal begins and ends gradually and causal as displayed on top of the figure.

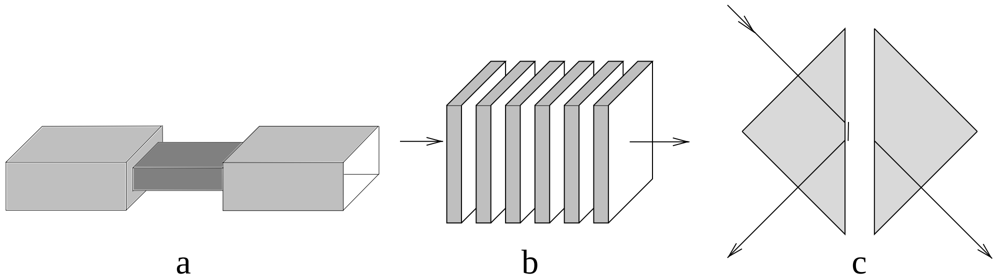


Fig. 3 Sketch of the three prominent photonic barriers. a) illustrates an undersized waveguide (the central part of the wave guide has a cross-section being smaller than half the wavelength in both directions perpendicular to propagation) , b) a photonic lattice (periodic dielectric hetero structure), and c) the frustrated total internal reflection of a double prism, where total reflection takes place at the boundary from a denser to a rarer dielectric medium.

important role in microwave technology and in classical optics. As a result of their mathematical analogy with wave mechanical tunnelling the instantaneous process of the spreading of evanescent fields is called photonic tunnelling [5, 6]. Three prominent photonic barriers for studying the tunnelling process are presented in Fig. 3.

The article begins with a presentation of the special properties of a physical signal. Amazingly enough in the following section it is shown that superluminal signal velocities can shorten the luminal time span between cause and effect but do not violate the principle of causality.

2 Signals

A signal is thought to cause a corresponding effect. Basically phonons, photons, and electrons are exploited to mediate interaction or to transmit bits, words or any desired signal to induce requested effects. The front or the discontinuous beginning of a signal is only well defined in the mathematical case of an infinite frequency spectrum. Any physical transmitter produces signals of finite spectra only. Therefore a front has no physical meaning and the narrow–band and the wide–band envelope (defined and discussed in Ref. [1, 20]) is the appropriate signal description. An example is displayed in Fig. 2a. In the case of an infinite signal spectrum the Planck quantum $\hbar\omega$ of the minimum energy of a field would necessarily result in an infinite signal energy as mentioned in the introduction.

A signal may be a single photon or electron with a distinct energy, more general a signal is characterized by a carrier frequency and its modulation and it is bound to be independent of magnitude otherwise we could not listen to the radio broadcasting or mobile phone during driving a car. There are frequency (FM) and amplitude (AM) modulated signals or a combination of both. A signal is not described by an analytical function, otherwise the complete information would be contained in the forward tail of a signal. In the latter case a detector could recognize from the forward tail the rest of the shape of the signal [21]. This becomes obvious for instance in digital optoelectronic communication systems, where measuring the signal half width gives the number of digits. Such a digital signal sent via glass fiber in modern communication systems is presented in Fig. 2(a).

In the same figure part 2(b) is a calculated sine wave signal of 10 oscillations plotted. The dotted one has an unlimited frequency–band width, whereas the solid one has a finite frequency–band. The carrier frequency is 5 GHz and the width 500 MHz. As a result of the frequency–band limitation there are noncausal components. The calculated result follows from the mathematical Fourier transform of a frequency band limitation. The noncausal components are not measurable. Any physical signal is beginning gradually at the time $t = 0$ as seen in Figs. 1, 2(a) [1, 9, 20]. A discontinuously signal beginning would make necessary the existence of infinite frequency components.

In the following we repeat the definitions of velocities which are found presented extensively elsewhere, e.g. in Refs. [1, 6, 7, 20, 22]:

$$v_\varphi = \omega/k \tag{1}$$
$$v_g = d\omega/dk \tag{2}$$
$$v_s \equiv v_g, \tag{3}$$

where ω and k are the angular frequency and the wave number, respectively. v_φ is the phase, v_g is the group, and v_s is the signal velocity. Relation (3) of the signal velocity v_s is valid in vacuum or in other anomalous dispersion–free media.

For the problem of signal propagation the following terms are used to describe the delay of the various parts of a signal envelope. The delay times have been analyzed for instance in the text book on Fourier Transform by Papoulis [20]:

$$t_\varphi(\omega) = \varphi(\omega)/\omega \tag{4}$$
$$t_g(\omega) = d\varphi(\omega)/d\omega \tag{5}$$
$$t_{fr}(\omega) = \lim_{\omega\to\infty} \varphi(\omega)/\omega \tag{6}$$

where t_φ is the phase time delay, t_g is the group time delay, and t_{fr} is the front time delay, see Fig. 4 [20]. $\varphi = kz$ is the phase in a medium at length z. The frequency–band $A(\omega)$, the phase angle φ, and the delay times are illustrated in Fig. 4. If φ does not tend to a straight line as ω tends to infinity, then the term signal front delay has no meaning [1, 20]. This behaviour takes place in the case of frequency–band limited signals in any medium and in the case of tunnelling, where the wave number is imaginary and thus the angle φ does not depend on length z. If the signal is narrow-band limited, there is no distortion of the envelope. Therefore the delay of the signal envelope is assumed to equal the delay of the center of gravity [20].

The classical *luminal forerunners* [1, 2, 22] don't exist in the case of a tunnelling signal containing evanescent modes only. Luminal forerunners exist only in dispersive media with a finite real refractive index.

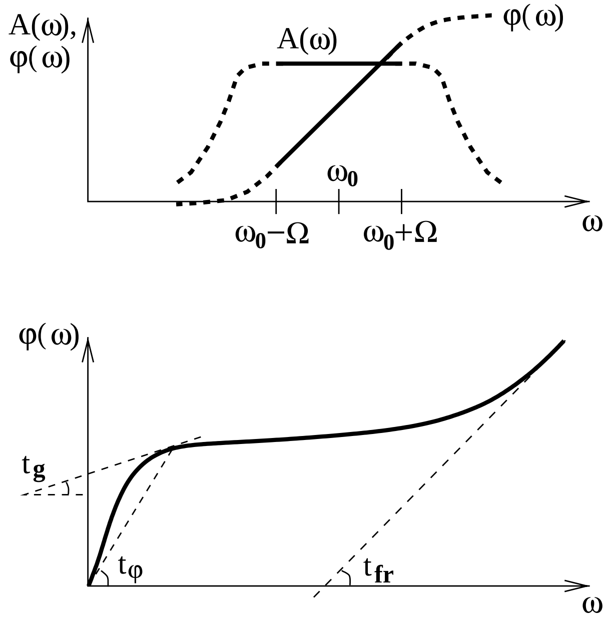


Fig. 4 A sketch of a signal's frequency-bandwidth $A(\omega)$ (central frequency $\omega_0 \pm \Omega$), of phase angle $\varphi(\omega)$, and of various delay times as defined e.g. in Ref. [20]. Ω is the frequency-band width $\omega_0 \pm \Omega$ around the center frequency ω_0.

3 Superluminal Signals

Since Helmholtz and Schrödinger equations are mathematically identical, it is evident that the three kinds of barriers displayed in Fig. 3 can be used to model the one–dimensional process of wave mechanical tunnelling [5, 6].

Some superluminal data obtained with the historical double–prism experiment are presented in the following [5, 6, 23, 24]. This smart experiment elucidates in a brilliant way the time behaviour of the tunnelling process [25].

Figure 5 indicates that for an angle of incidence greater than the angle of total reflection the barrier transmission time of the double–prism, or what we call here the tunnelling time can be split into two components

$$t_{\mathrm{tunnel}} = t_{\|} + t_{\perp}, \tag{7}$$

one along the surface due to the Goos-Hänchen shift D, and another part perpendicular to the surface [23, 25]. The measured tunnelling time t_{tunnel} represents the group time delay which results in the group or signal velocity (see Eq. (3)).

The first component of t_{tunnel} is related to a non-evanescent wave characterized by the real wavenumber

$$k_{\|} \;:=\; k_0\, n_1\, \sin\theta_i \tag{8}$$

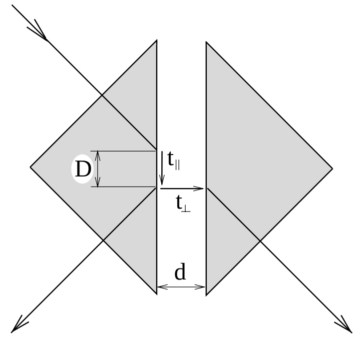


Fig. 5 The tunnelling time of the double-prism experiment consists of two components. $t_{\|}$ for the Goos–Hänchen shift D parallel to the prism's surface and $t_{\perp}$ for crossing the gap in the direction perpendicular to the two surfaces of the gap.

while the second one

$$k_{\perp} := i\, k_0 \sqrt{n_1^2 \sin^2 \theta_i - 1} \tag{9}$$

is related to the evanescent mode traversing the gap between the two prisms. $k_0 = 2\pi/\lambda_0$, where λ_0 is the corresponding vacuum wavelength, and n_1 the refractive index of both prisms. Details of the experiment are given in [23].

In the symmetrical design of the experiment displayed in Fig. 5 the reflected and the tunnelled signal leave the first and the second prisms at the same time. This result represents an experimental proof that for the tunnelling time component $t_{\perp}$ holds $t_{\perp} = 0$. It is in agreement with the observations on other photonic tunnelling structures: The finite measured tunnelling time accrues at the entrance boundary and no time is spent inside a barrier.

A couple of months after the discovery of superluminal tunnelling of microwave signals [26] a study on superluminal group velocity and transmission of single optical photons tunnelling a photonic barrier was published [27]. Yes certainly, as the authors claimed they did not measure a signal velocity as the photons were emitted in a spontaneous process. However, the group velocity of the investigated black box (i.e. the tunnelling barrier) has been determined and the data are also valid for the transmission of signals. Sending signals containing millions of optical photons analogous to the microwave experiment, the black box would result in the same superluminal group velocity as in the single photon experiment. In fact the same experimental set-up and procedure have been proven with a sample of bulk glass instead of the tunnelling barrier this black box analogy to be correct [28]. Actually in this case the single photon experiment yielded the *sub*-luminal group velocity known from bulk glass as measured in standard spectroscopy. In both in the microwave and in the single photon experiments the group velocity has been measured with a detector located in free space far away from the investigated black box. In such asymptotic measurements the relation (3) holds, i.e. the group velocity equals the signal velocity.

4 Does Superluminal Signals Violate The Principle Of Causality?

Photonic tunnelling experiments have revealed superluminal signal velocities [19, 23, 26] as well as superluminal group and energy velocities [27] of single photons. According to textbooks a superluminal signal velocity violates Einstein causality implying that cause and effect can be changed and time machines known from science fiction could be constructed. This interpretation, however, assumes a signal to be a point in the time dimension neglecting its finite duration. Can a signal travelling faster than light really violate the principle of causality stating that cause precedes effect [29]? The latter question has been widely assumed as a matter of fact and it has allegedly been shown that a signal velocity faster than light allows to change the past. The line of arguments how to manipulate the past in this case is illustrated in Fig. 6 [3, 10, 11, 12, 13]. There are two frames of reference displayed. In the first one at the time $t = 0$ lottery numbers are presented, whereas at $t = -10$ ps the counters were closed. Mary (A) sends the lottery numbers to her friend Susan (B) with a signal velocity of 4 c. Susan, moving in the second inertial system at a relative speed of 0.75 c, sends the numbers back at a speed of 2 c, which arrives in the first system at $t = -50$ ps, thus in time to deliver the correct lottery numbers before the counters close at $t = -10$ ps.

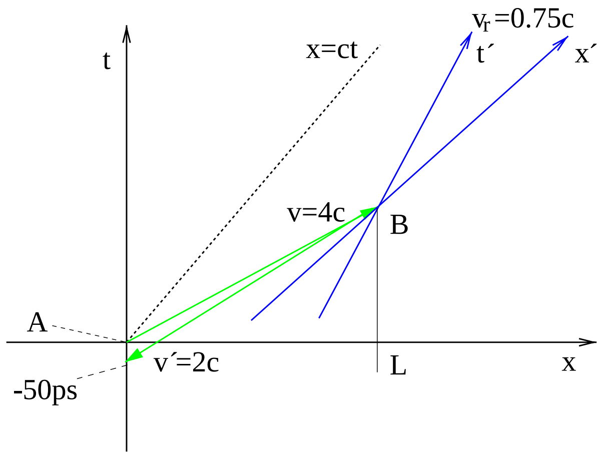


Fig. 6 Coordinates of two observers **A** $(0, 0)$ and **B** with $O(x, t)$ and $O'(x', t')$ moving with a relative velocity of 0.75 c. The distance L between **A** and **B** is 0.1 m. **A** has available a signal velocity $v_s = 4$ c and **B** $v'_s = 2$ c. The numbers in the example are chosen according to [29]. The signal returns -50 ps in the past in **A**.

The time shift of a point on the time coordinate of the reference system A into the past is given by the relation [11, 29]:

$$t_A = -\frac{L}{c} \cdot \frac{(v_r - c^2/v_s - c^2/v'_s + c^2 v_r/v_s v'_s)}{(c - c v_r/v'_s)} \tag{10}$$

where L is the transmission length of the signal, v_r is the velocity between the two inertial systems A and B. The condition for the change of chronological order is $t_A \leq 0$, the time shift between the systems A and B.

In the experiments displayed in Fig. 1 the signals travelled at a superluminal velocity with $-c/310$ [18] and with 4.7 c [19] , respectively. Nevertheless, the principle of causality has not been violated in both experiments. In the example with the lottery data the signal was assumed to be a point on the time coordinate. However, a signal has a finite duration as the pulses sketched along the time coordinate in Fig. 7.

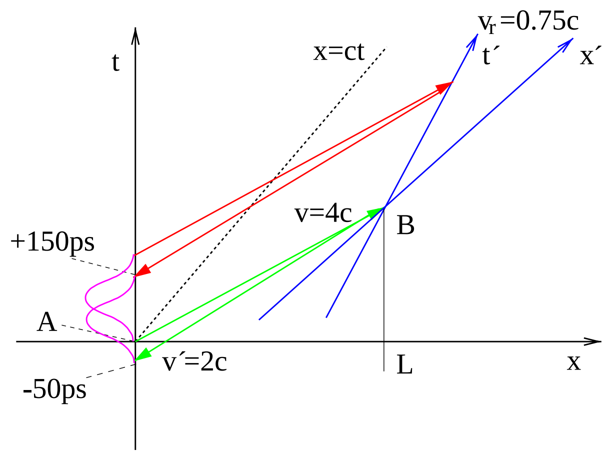


Fig. 7 In contrast to Fig. 6 the pulse has now a finite duration of 200 ps. This data is used for a clear demonstration of the effect. In both experiments, the signal length is long compared with the measured negative time shift in spite of the superluminal signal velocity. A signal envelope ends generally in the future.

The two signals presented in Fig. 1 have lengths of 7 μs and 4 ns. Any signal like a bit, a word or a sentence has a finite extension on the time coordinate. Assuming in our example a signal's finite duration of 200 ps the complete information is obtained with superluminal velocity but only at positive times under the assumptions illustrated in Fig. 7. The same holds a fortiori for the two presented experiments. The finite duration of a signal is the reason that a superluminal velocity does not violate the principle of causality. A shorter signal with the same information (such a transform is done frequently in modern transmission technology) has an equivalently broader frequency band. In this case as a result of the dispersion relations of tunnelling barriers or of any interaction a pulse reshaping would take place. Such a pulse reshaping is demonstrated in Fig. 8, details are presented in Ref. [30].

The pulse reshaping is a result of a discontinuous leading front and an unlimited frequency–band. Both shape and half width are changed during the tunnelling process.

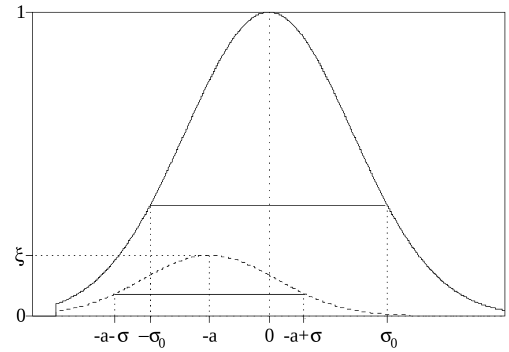


Fig. 8 Comparison of normalized intensity vs time of an air born signal (solid line) and a simulated tunnelled signal (dotted line) moving from right to left. The signals have a discontinuous beginning and the frequency band is unlimited. The tunnelled pulse is attenuated and reshaped. Moreover, although its maximum has travelled at superluminal speed, both fronts have traversed the same distance with the light velocity in vacuum. Here ξ is the maximum of the tunnelled pulse, a is the shift of the maximum, σ is the variance of the tunnelling signal, and σ_0 is the variance of the air born pulse. It is obvious that the latter is longer than the variance of the tunnelled pulse.

5 Summing up

Evanescent modes show some amazing properties to which we are not used to from classical physics. Apparently evanescent modes are nonlocal fields and represented by virtuell photons [31]. Evanescent modes are easily traceable through barriers some 100 wavelengths thick and they do not spend time in the barrier. The latter is an experimental result due to the fact that the transmission time is independent of barrier length (Hartman effect [32, 33, 34]). Another proof of this behaviour is observed in the case of symmetrical Frustrated Total Internal Reflection (FTIR), where the reflected and the transmitted signal have the same delay time, i.e. the time spent inside the barrier is zero. The measured finite transmission time comes into existence at the entrance boundary of the photonic barriers.

In conclusion the principle of causality has not been violated by superluminal signals as a result of the finite signal length and the corresponding frequency-band width. But amazingly enough the time span between cause and effect is reduced by a superluminal signal velocity compared with luminal cause to effect propagation.

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References

[1] L. Brillouin, *Wave propagation and group velocity*, Academic Press, New York (1960)
[2] G. Nimtz and W. Heitmann, Prog.Quantum Electronics **21** (1997) 81
[3] R. U. Sexl and H. K. Urbantke , *Relativity, Groups, Particles* (Springer, Wien, NewYork 2001)
[4] A. Einstein, Ann.Phys. (Leipzig) **17** (1905) 891
[5] A. Sommerfeld, *Vorlesungen über Theoretische Physik*, Band IV, Optik, Dieterich'sche Verlagsbuchhandlung (1950)
[6] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feyman Lectures on Physics*, Addison–Wesley Publishing Company, **II** 33–12 (1964)
[7] E. Merzbacher, *Quantum Mechanics*, 2nd ed., John Wiley & Sons, New York (1970)
[8] G. Nimtz, Ann. Phys. (Leipzig) **7** (1998) 618
[9] G. Nimtz, Eur. Phys. J. B. **7** (1999) 523
[10] R. Chiao and A. Steinberg, Progress in Optics, **XXXVII** (1997) 345
[11] R. Sexl und H. Schmidt, *Raum-Zeit-Relativität*, vieweg studium, Braunschweig (1978)
[12] H. Goenner, Ann.Phys. (Leipzig) **7** (1998) 774
[13] E. Recami, Int. J. of Modern Physics, **A 15** (2000) 2793
[14] M. Büttiker and H. Thomas, Supperlattices and Microstructures, **23** (1998) 781
[15] C. E. Shannon, Bell Syst. Tech. J., **27** (1948) 379;
C. E. Shannon, Proc. IRE, **37** (1949) 10
[16] Hewlett Packard, *Spectrum Analysis Application Note* 150 (1989)
[17] R. L. Smith, Am. J. Phys., **38** (1970) 978
[18] L. J. Wang, A. Kuzmich, and A. Dogariu, Nature, **406,** (2000) 277
[19] G. Nimtz, A. Enders, and H. Spieker, J. Phys. I., France **4** (1994) 565
[20] A. Papoulis, *The Fourier Integral And Its Applications*, McGraw-Hill, New York (1962), Sec. 7.5 and 7.6
[21] F. Low, Ann. Phys. (Leipzig) **7** (1998) 660
[22] J.A.Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company, Inc. New York and London (1941)
[23] A. Haibel, G. Nimtz, and A. A. Stahlhofen, in press : Phys. Rev. E, March (2001)
[24] A. Haibel and G. Nimtz, Ann. Phys. (Leipzig) in press (2001)
[25] A. A. Stahlhofen, Phys. Rev. A, **62** (2000) 12112
[26] A. Enders and G. Nimtz, J. Phys. I (France) **2** (1992) 1693
[27] A. Steinberg, P. Kwiat, and R. Chiao, Phys. Rev. Letters, **71** (1993) 708
[28] A. Steinberg, P. Kwiat, and R. Chiao, Phys. Rev. Letters, **68** (19992) 2421
[29] P. Mittelstaedt, Eur. Phys. J. B, **13** (2000) 353
[30] Th. Emig, Phys. Rev. E **54** (1996) 5780
[31] C. K. Cargnilia and L. Mandel, Phys. Rev. D **3** (1971) 280
[32] Th. Hartman, J. Appl. Phys. **33** (1962) 3427
[33] A. Enders and G. Nimtz, Phys. Rev. E **48**, (1994) 632
[34] Ch. Spielmann et al., Phys. Rev. Letters **73** (1994) 2308